

# Optimal siting of DG units in power systems from a probabilistic multi-objective optimization perspective



Payman Dehghanian<sup>a</sup>, Seyed Hamid Hosseini<sup>a,\*</sup>, Moein Moeini-Aghtaie<sup>a</sup>, Amirsaman Arabali<sup>b</sup>

<sup>a</sup> Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

<sup>b</sup> Department of Electrical and Biomedical Engineering, University of Nevada, NV, USA

## ARTICLE INFO

### Article history:

Received 14 July 2012

Received in revised form 3 February 2013

Accepted 22 February 2013

Available online 25 March 2013

### Keywords:

Distributed generation (DG)

Placement

Multi-objective (MO)

Non-dominated Sorting Genetic Algorithm (NSGAI)

Power distribution system

## ABSTRACT

Along with the increasing demand for electrical power, distributed generations (DGs) have so far found their pivotal roles in the restructured environment of power distribution systems. As an indispensable step toward a more reliable power system, the DGs optimal allocation strategy, deemed to be the most techno-economically efficient scheme, comes to the play and is profoundly taken under concentration in this study. This paper devises a comprehensive multi-objective (MO) optimization approach by which all the crucial and maybe contradictory aspects of great influence in the placement process can be accounted for. Total imposed costs, total network losses, customer outage costs as well as absorbed private investments are those considered objectives in the proposed scheme. Non-dominated Sorting Genetic Algorithm II (NSGAI), as a robust widely-used method of multi-objective dilemmas, is employed to cope with the optimization problem. Point Estimation Method (PEM) has also lent the authors a hand in probabilistically approaching the involved uncertain criteria. In the light of the proposed methodology being implemented on the 37-Bus IEEE standard test system, the anticipated efficiency of the proposed method is well verified.

© 2013 Elsevier Ltd. All rights reserved.

## 1. Introduction

### 1.1. Motivation and problem description

Technical, economical, and environmental concerns for more reliable and economic sources of power generation have led to the ever-increasing interests toward the distributed generation (DG) schemes [1]. DGs seem to fully be able to come to grips with the huge amount of investment costs associated with the system upgrading and planning [1]. On the other hand, facing the considerable appearance of new loads in power systems, as well as peak load demand growth, enlighten the way to DGs common acceptance in power systems [2].

The type and scope of embedded generation differs significantly from country to country. The difference, however, may also arise due to the different policies adopted for commercial influences. Among the available technologies of embedded generations are fuel cells, combined heat and power (CHP), wind generations, micro-turbines, hydro turbines, and photovoltaic (PV) devices [3].

By right, and irrespective of the various technologies and strategies, some technical and economic implications would be consequential due to the DG penetration into power systems.

Technically reported, the flow of both real and reactive powers may need to be reorganized due to the congestion problem [4]. As a consequence, active and reactive power losses would be modified and voltage profile variations may be experienced [5]. During the last decade, DG's attractive features and wide influences on power system protection [6], and power system stability [7] have been under the immense explorations. On the other hand, the restructuring trend in power system along with the current open door to the power market has, so far, introduced a fact that the non-technical impacts of DG penetration in power distribution systems need to be seriously addressed, more from an economic point of view [8]. The costs associated with the power losses, congestions, and emission would be, therefore, the main targets [9].

### 1.2. Literature survey

Planning, design, and operation of power systems in presence of DGs have recently been an attractive ongoing area of research which has led to some tremendous number of projects, reports, and papers. The challenges associated with DG allocation can be considered as a matter of interest in two contexts; not only the placement concerns and objectives have been of great curiosity,

\* Corresponding author. Address: Department of Electrical Engineering, Sharif University of Technology, P. O. Box 11155-4363, Tehran 11155, Iran. Tel.: +98 21 66165932; fax: +98 21 66023261.

E-mail addresses: [Payman\\_Dehghanian@ee.sharif.edu](mailto:Payman_Dehghanian@ee.sharif.edu) (P. Dehghanian), [hosseini@sharif.edu](mailto:hosseini@sharif.edu) (S.H. Hosseini), [Mmoeini@ee.sharif.edu](mailto:Mmoeini@ee.sharif.edu) (M. Moeini-Aghtaie), [Saman\\_Arabali@yahoo.com](mailto:Saman_Arabali@yahoo.com) (A. Arabali).

## Nomenclature

### A. Variables

$A_t$	periodic revenue.
$\overline{CF}_{DG,i}$	capacity factor of DG at the $i$ th bus
$EENS_i$	expected energy not served in the $i$ th bus
$f_i^{new} \forall i \in [1-4]$	the modified objective functions used in the constraint handling process of the optimization problem under study
$g_i \forall i \in [1-3]$	the constraints violation variables used in the optimization problem under study
$\overline{MP}$	expected value of market price
$n_i$	number of new added DG units at the $i$ th bus
$P_i$	net real power of the $i$ th bus
$P_{DG,i}$	output real power of DG at the $i$ th bus
$P_{DG,i}^{nom}$	nominal capacity of DG at the $i$ th bus
$P_{D,i}$	load of the $i$ th bus
$pf_i \forall i \in [1-3]$	the penalty factors used in the constraint handling process of the optimization problem under study
$Q_{DG,i}$	output reactive power of DG at the $i$ th bus
$Q_i$	net reactive power of the $i$ th bus
$r$	rate of return on an investment.
$RoR_{DG,i}$	mean value of the rate of return associated with the DG at the $i$ th bus
$U_i$	annual outage time of the $i$ th load point
$V_i$	voltage magnitude of the $i$ th bus
$X$	vector of random input variables in the probabilistic analysis
$Y$	vector of random output variables in the OPF problem
$mu_Y$	vector of expected value associated with the random output variables
$\sigma_{DG,i}$	investment risk of DG installment at the $i$ th bus
$\sigma_Y$	vector of standard deviation values associated with the random output variables

### B. Parameters

$C_i$	capital cost of a predefined-size DG
$CC_i$	capital cost of DG at the $i$ th bus (\$/kV A)
$COC_{base}$	base case customer outage cost of distribution network
$COC_i$	customer outage cost of distribution network regarding the modified load amount of $i$ th bus
$COCS_i$	customer outage cost sensitivity factor of the $i$ th bus

$d$	discount rate in the planning horizon
$IC$	initial investment cost
$IEAR_i$	interrupted energy assessment rate of the $i$ th load point
$L_{a,i}$	average load of the $i$ th bus
$LS_i$	loss sensitivity factor of the $i$ th bus
$n$	number of input random variables in PEM
$OC$	operation cost of DG (\$/kV A)
$P_{DG,i}^{max}$	maximum possible real power of the installed DG at the $i$ th bus
$P_{DG,i}^{min}$	minimum possible real power of the installed DG at the $i$ th bus
$Q_{DG,i}^{max}$	maximum possible reactive power of the installed DG at the $i$ th bus
$Q_{DG,i}^{min}$	minimum possible reactive power of the installed DG at the $i$ th bus
$R_{ij}$	line resistance between bus $i$ and $j$
$RoR_D$	desired level of rate of return
$S_i$	sensitivity factor of the $i$ th bus
$T$	planning horizon of the studies
$TL_{base}$	total network losses in the base case
$TL_i$	total losses of distribution network with modified amount of load at the $i$ th bus
$U_i$	undelivered energy associated with the $i$ th bus
$V_i^{max}$	maximum allowable voltage magnitude of the $i$ th bus
$V_i^{min}$	minimum allowable voltage magnitude of the $i$ th bus
$W$	present value of salvage cost
$\alpha$	loss sensitivity factor share
$\beta$	customer outage cost sensitivity factor share
$\delta_i$	voltage angle of the $i$ th bus
$\sigma_L$	maximum allowable level of project risk
$\sigma_{X,k}$	standard deviation of the $k$ th input random variable
$\mu_{dt}$	satisfactory level for the membership functions of the $i$ th objective function
$\mu_{fi}$	decision maker degree of truth for the $i$ th objective function
$\mu_{X,k}$	mean value of the $k$ th input random variable
<b>C. Sets</b>	
$N$	set of network buses

but also the approaches and access ways toward the problem, their optimality, and accuracy have been long under investigation in the literature [10].

As far as the authors' knowledge, a few effective and implementable works have been conducted in incorporating all the essential factors and objectives, technical sides as well as the economic aspects interrelated with the DG placement problem. Most of the existed literature on the subject under consideration have been just concentrated on single objectives in their evaluations; some works have been done on DG placement taking into account system losses solely as their main target [9]. Some references are devoted to the problem just from system reliability improvement viewpoint [11–13]. Voltage profile is the other interest target among some authors [14]. A cost/worth analysis approach is introduced in [15] to have the DGs optimally connected to the electric network.

Some techniques and algorithms have been employed or devised in the literature for the sake of DG allocation. Amongst are Genetic Algorithm (GA) [16–23], Particle Swarm Optimization [20,24] Ant Colony Optimization [25], Evolutionary Programming [26], and some other heuristic approaches [10,27]. These types of

approaches have some positive and also negative characteristics which have been well explored in [10].

Exploring the literature with emphasis on the problem objectives, there are also some papers which are concentrated on the multi-objective treatment of the DG siting problem [10,28–30]. However, almost none of them has considered the economical criterion, i.e., absorbed private investment cost, together with the other technical constraints once deciding about the sizing and siting of DG units in distribution systems. Also, most of them have not utilized an appropriate approach to deal with the contradictory objectives, if any. They, in addition, cannot guarantee reaching to the global optimum solutions or have major problems in constrained optimizations and their convergence speeds, or suffer from a computational burden which restricts their adoptions in practical applications. Those remaining ones which utilized the Genetic Algorithm to solve the problem either have not considered both the technical and economical objectives together [16,21–23], or have not delved into the probabilistic treatment of the uncertain and stochastic factors existed in the decision making under study [31,32].

### 1.3. Paper targets and contribution

The DG placement problem is attacked as an effective alternative of distribution network reinforcement. This paper attempts to accomplish the goal of contributing all these essential factors in a well-organized MO optimization approach which takes care of both the technical and economic aspects. Here, the necessity of considering the total losses of distribution network, together with the investment, operation, and maintenance costs (total imposed cost) of new DG units is entirely investigated. The reliability worth and the investors' profit maximization as the other two objectives lend the authors a hand to profoundly study the problem.

In order to deal with these contradictory and inevitable objectives and criteria in this process, a robust multi-objective optimization method should be employed. The multi-objective optimization approach has been introduced as an appropriate tool for handling incommensurable objectives with conflicting/supporting relations. As the problem dimension increases, it, however, becomes infeasible to reach an optimal solution in which all the objectives can be optimized. Different methods are introduced in literature to deal with such kinds of multi-objective optimization problems which can be classified as priori or posterior methods [33,34].

In priori methods such as weighted sum,  $\epsilon$ -constraint method, a relative preference vectors should be supplied without any knowledge of possible consequences. Therefore, the preference vector may come to a suboptimal result or an infeasible one. This is highly subjective to the decision maker, too. In contrast, the posterior methods try to find the Pareto solutions applying non-dominancy concepts. All these optimal solutions can be fed to a decision making process in order to find the final optimal solution considering the decision makers' preferences [33,34].

NSGAI is commonly approved and widely utilized once dealing with the multi-objective optimization problems [35–38]. The reason lies mainly in the fact that it has a strong ability in finding the global optima of any multi-objective optimization problems in comparison with the other approaches and this capability has been credibly proven in the literature [37]. Understanding the ability and superiority of the NSGAI in treating the multi-objective optimization problems compared to other approaches can easily lead to the final conclusion that the results obtained via the proposed method would be definitely the most optimized ones, possible ever.

Based on the above discussions and to fill the aforementioned gaps, the main contributions of the paper can be considered in three different aspects.

- Multi-Objective Treatment of the Problem Using the New and Effective Critical Objectives.

The presented paper serves as one of the pioneers in dealing with the DG placement problem not only from the technical viewpoints (loss reduction, power system reliability), but also from the economical perspectives (investment and operation costs together with the absorbed private investors' profit maximization which has not been studied previously in the placement cases). The authors believe that the private investors' concerns have also to be considered in the placement procedure of distributed generation since in the nowadays open access environment of power systems, the private investors are expected to participate in these planning studies.

- Stronger Optimization Technique Utilized to Guarantee the Global Optima of the Problem.

The proposed algorithm also utilizes a stronger and also modified/updated version of the conventional Genetic Algorithm, i.e., NSGAI, whose superiorities have been well confirmed in different cases of engineering optimizations.

- Probabilistic Treatment of the Existent Stochastic and Uncertain Factors of the Problem.

The proposed scheme in this paper has also fell into the probabilistic treatment of the uncertain and stochastic factors interrelated to the decision making problem under study (stochastic nature of power system loads together with the existent variation in the market prices of electrical energy). In so doing, a very effective and efficient approach, i.e., the 2-Point Estimation Method (2-PEM), has lent the authors a hand to get the most out of the proposed scheme. The proposed algorithm is thereafter justified being implemented on the IEEE standard 37-bus test system.

### 1.4. Paper organization

The rest of the paper is organized as follows. Section 2 is comprised of three main parts; the first part discusses the four objectives incorporated in the DG placement procedure in this paper; the second part is devoted to the subject of uncertainties modeling in power systems; and the third part presents the PEM approach employed in this paper. Section 3 introduces the NSGAI algorithm as a robust widely used approach in the cases of MO optimization problems. It also elaborates on a fuzzy-based decision making method to get the final decision out of the optimization framework. Section 4 presents the proposed algorithm and sets forth the methodology of this paper. Section 5 involves in the implementation process of the proposed technique through a case study and also discusses the obtained results. Finally, the conclusions are outlined in Section 6.

## 2. Problem statement and formulation

This section discusses the pivotal issues associated with the DG siting problem in a restructured environment. These issues include the necessity of considering different criteria and the process of uncertainties modeling. The formulation of objective functions is also presented in this section.

### 2.1. Objective functions

Many distribution companies (Discos.) around the world have faced this challengeable subject, i.e., DG allocation in power systems, whose pros and cons motivate them to avidly investigate the profitability of this technology. Its encouraging consequences in distribution network losses, voltage profile, less dependency to the electrical energy market price, and reliability improvements through the resulted drop off in outage duration and frequencies are among these incentives that encourage the Discos to anticipate more economical profitability by then. Hence, a strategy is needed be properly designed to cover all the above considerations so that it would help the utility meet its technical and financial targets. The objectives associated with the planning process of DGs in distribution networks can play an important role in accordance with the utility perspectives. The proposed scheme is founded on the basis of four main technical–economical objectives. Total imposed costs (investment, operation, and maintenance costs) and total network losses are among the factors which are needed to be considered since they are the most important ones in the view point of distribution operators who want to be techno–economically satisfied by executing the DG placement schemes in the network. In

order to propose a practical algorithm which can cover the customers' concerns as well, we have introduced the customer outage cost as the other appropriate criterion. To reach some plans which are attractive also in the viewpoint of private investors, their concerns are also modeled so that the private investment can be maximized. Thus, the objectives are introduced as follows.

### 2.1.1. Total imposed costs

Investment cost is observed to be of the major driving forces behind any investment opportunity. Many constraints such as the Disco's annual budget, different reinforcement schemes, and their annual expenditures have led to the total imposed cost being considered as a classical objective in the cases of planning problems [8,12–14]. The total imposed cost to the utility to be minimized is comprised of annual investment costs and operation costs of the embedded generations, as shown in the following equation:

$$f_1 = \min \sum_{t=1}^T \sum_{i=1}^N \left\{ \frac{1}{(1+d)^t} \cdot CC_i \cdot P_{DG,i}^{nom} + \left( \frac{1+i}{1+d} \right)^t \cdot P_{DG,i}^{nom} \cdot \overline{CF}_{DG,i} \cdot OC \cdot 8760 \right\} \quad (1)$$

The electrical energy pricing frameworks in the restructured environments are of significant consequences to the profit-making performance of DG owners. Distribution network operation and planning strategies, accompanied by the adopted pricing policies cause a considerable influence on the amount of DGs power production level [1]. In other words, the existent uncertainties in the electricity market price can well affect the owners' revenue. The reason lies in their revenue dependency to the DGs outputs in power distribution systems. These variations in DGs output power are modeled via the Capacity Factor (CF) considered in the DGs operation costs. In response, the electricity market price (and accordingly the CF variations) is modeled applying a robust approach in dealing with the probabilistic problems referred to as the Point Estimation Method (PEM) which is going to be addressed later in this paper.

### 2.1.2. Total network losses

Obviously, any loss reduction in the power system is of considerable contribution to meet the better performance of distribution utilities. A small penetration amount of a strategically allocated DG can cause a significant reduction in the losses of vulnerable feeders [9]. When an embedded generator is located near to a large load, the network losses will be curtailed in consequence to the fact that the load is able to be fed by both real and reactive power from the adjacent generator. Conversely, a large embedded generator placed far away from the load centers would likely lead to the loss increase in distribution network [9,39]. It is of great importance to note that a further complication appears facing the network with the ever-increasingly growing demands and complexities associated with the network expansion. Hence, load levels and DG capacities together with their locations in the distribution network are mainly the factors of great influences on the network real power losses [9,39]. In short, DG penetration can cause major changes in voltage magnitudes and accordingly power flows. These changes will well affect power system losses. Even though the losses cannot be entirely removed, they can be brought down to an acceptable level. However, DG installations at non-optimal locations can evenuate in a significant increase in system losses, triggering an increase in costs and, hence, has some negative impacts on the utility's desired target. Loss reduction is, therefore, the most important factor which needs to be considered in the DG placement problem. In this paper, this objective function is mathematically written as follows [9,39].

$$f_2 = P_{loss} = \sum_{i \in N} \sum_{j \in N} A_{ij} (P_i P_j + Q_i Q_j) + B_{ij} (Q_i P_j - P_i Q_j) \quad (2.a)$$

$$A_{ij} = \frac{R_{ij} \cos(\delta_i - \delta_j)}{V_i V_j} \quad B_{ij} = \frac{R_{ij} \sin(\delta_i - \delta_j)}{V_i V_j} \quad (2.b)$$

s.t.

- Power balance constraint:

$$\sum_i P_{DG,i} = \sum_i P_{D,i} + P_{loss} \quad (2.c)$$

- Voltage limits:

$$|V_i^{\min}| \leq |V_i| \leq |V_i^{\max}| \quad (2.d)$$

- Real power generation limits:

$$P_{DG,i}^{\min} \leq P_{DG,i} \leq P_{DG,i}^{\max} \quad (2.e)$$

- Reactive power generation limits:

$$Q_{DG,i}^{\min} \leq Q_{DG,i} \leq Q_{DG,i}^{\max} \quad (2.f)$$

### 2.1.3. Customer outage cost

Certainly, customers' needs and willingness to pay for reliability is of the crucial factors to be considered in the planning studies of a Disco. The continuity of delivered energy to customers directly affects the profitability of a utility [40]. On the other hand, the reliability level enhancement of distribution grid ordinarily calls for some installations of new equipment which generally triggers a fundamental conflict with the classical planning concern (minimization of total imposed costs). Fig. 1 illustrates this conflict and the worth of providing reliable services. In essence, the total earned profit of utility is simultaneously a function of the investment cost and customer outage cost. As Fig. 1 shows, the reliability level enhancement of a distribution feeder mainly results in an increasing consequence of the company costs. On the other hand, as the reliability level goes up customer willingness to pay decreases [40]. This illustrates that a considerate compromise has to be conducted and the two basic factors (company cost and customer willingness to pay) determine the optimum reliability level.

It has been shown that the DG allocation in distribution networks would lead to further attainment of reliability improvement [11]. To deal with the quality and continuity of electrical energy, the reliability level has been usually considered as a constraint in the planning problems. This viewpoint has been successfully used in traditional environments; however, it is unable to weigh up the systems' overall economic losses during system operation in a

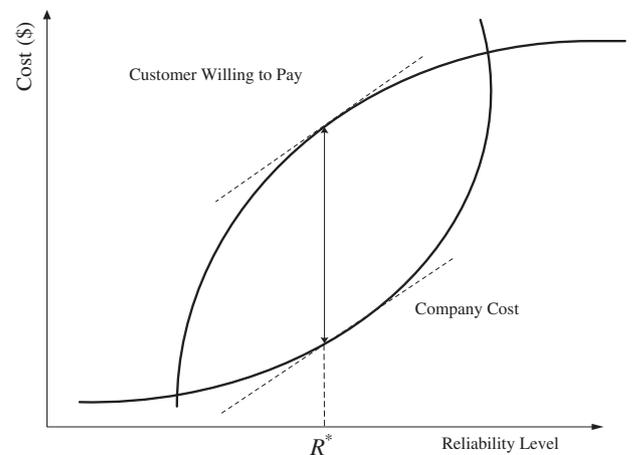


Fig. 1. Reliability cost and reliability worth relationship.

competitive market. Regarding the reliability criterion as an objective function would guarantee the existence of optimal plans [40] which are much more close to the apex point of the total profit. Hence, the customer outage cost is considered as an objective to be minimized in the DG planning procedure. This objective is determined by (3).

$$\min \sum_{i \in N} C_i \cdot n_i \quad (3.a)$$

s.t.

$$EENS \leq EENS_D \quad (3.b)$$

$$EENS = \sum_{i=1}^N L_{a,i} U_i \quad (3.c)$$

where  $EENS$  and  $EENS_D$  respectively denote the expected energy not supplied of the distribution network and the desired level of network reliability. In this respect, the optimal plan is undeniably dependent on the value of  $EENS_D$  whose determination process is very difficult and highly sensitive. As can be seen in Fig. 1, the total profit of utility as a function of the desired level of reliability could be ranged from the optimal point with regard to the different  $EENS_D$  values. This is due to the fact that in the planning procedures, the obtained results (as optimal plans) would get the minimum pre-determined reliability level ( $EENS_L$ ) so as to confirm the minimum imposed investment cost. So the modified objective is determined by:

$$f_3 = \sum_{i \in N} EENS_i \times IEAR_i \quad (4.a)$$

s.t.

$$EENS \leq EENS_L \quad (4.b)$$

The  $EENS_L$  has been used to ensure the minimum reliability level of distribution network. As a constraint, this level should be kept satisfied for all the optimal and non-optimal plans. Also, it is worth mentioning that the random behavior of power systems and consequently the uncertainties reflection in the customer outage cost are truly of the main causes of  $EENS$  consideration as the under-focused index of reliability in this paper [40].

#### 2.1.4. Absorbed private investment maximization

De-monopolization, absorption of the private investors and owners, and the free access for all investors can be accounted as the main principles of deregulation in power systems. One of the most important dilemmas in a deregulated power system is the unwillingness of private investors to invest in the costly projects of installing new DGs [1,2]. Lack of economic incentives and the uncertainties associated with the cost recovery of a project are areas of great apprehension [41]. In this respect, since the energy usages and the associated revenues are stochastic in nature, applying an influential probabilistic method allows the investors to determine the most opportunistic time to well start a project and acquire the maximum revenue returns by then. Also, the projects with insufficient economic incentives as well as the profitable ones can be, so, distinguished. On the other hand, as the candidate projects' rates of return and the involved risk are the most important factors for private investors, properly modeling of these factors helps a Disco manager to determine a series of economic incentives for absorption of private investments in distribution networks. Moreover, it allows the private investors to select and validate the package of incentives required to support new investment in response to the aging and loading conditions of the distribution network.

Consequent to the above discussions, to be able to comprehensively investigate the capital rate of return and the investment risk of different DG projects in the viewpoint of private investors, the

financial signals including both efficiency improvements and risk declination have to be carefully put under consideration as the long-term purposes of the responsible Disco. So, considering the maximization of the private investments for integrating DG units as an objective to contribute the vital economic signals seems unavoidable. The private investment absorption objective is introduced by the following equation:

$$f_4 = \sum_{i \in N} CC_i \cdot n_i \quad (5)$$

The existent uncertainties in a power system such as the predicted amount of future load and the volatility in the forecasted electrical energy price reform the DG units' production as a stochastic variable. Therefore, the revenue and rate of return derived from DG units is probabilistic since they are both functions of DG power output. As a result, investing in DG projects could be disposed to some unavoidable risks essentially due to its probabilistic incomes and rate of returns.

Portfolio theory as a well-developed paradigm, proposed by Markowitz in 1952, is an approach based on mean and variance analysis for selecting the economic portfolio among the at hand risky projects [42]. Respectively, the mean value of investment return rate and its deviation are considered as the mean and variance. The main goal of portfolio theory is to present a mathematical method for analysis and evaluation of economical portfolio choices based on risk-return trade-offs. According to the portfolio theory, a choice which results in higher expected return and lower risk has to be preferred as the first priority. This is truly in line with the fact that the investors are inclined to invest more capitals to increase their profits and return rates. It is worth mentioning that according to Tesler's criterion, preferring a risky project with higher rate of return subject to certain level of value at risk may be more pleasant [43]. During the useful life time of investment in DG plans, its rate of return can be obtained as follows.

$$A_t^{DG,i} = P_{DG,i}^{nom} \cdot \overline{CF}_{DG,i} \cdot \overline{MP} \cdot 8760 \quad (6)$$

$$\sum_{t=0}^T \frac{A_t}{(1+r)^t} = IC - W \quad (7)$$

The cash flow diagram of an investment is shown in Fig. 2. To consider the uncertain rate of return value, the rate of return mean value ( $RoR_{DG,i}$ ) and its standard deviation ( $\sigma_{DG,i}$ ), so called investment risk, are employed to identify the attractive DG projects. To this end, this paper introduces financial constraints in line with the projects' attractiveness. In this response, the rate of return and the risk level of each candidate plan for DG placement problem need to be checked in accordance with those of desired. The constraint satisfaction should be guaranteed as a necessary principle for each candidate. These economic constraints are defined as shown in (8.a) and (8.b).

$$\sigma_{DG,i} \leq \sigma_L \quad (8.a)$$

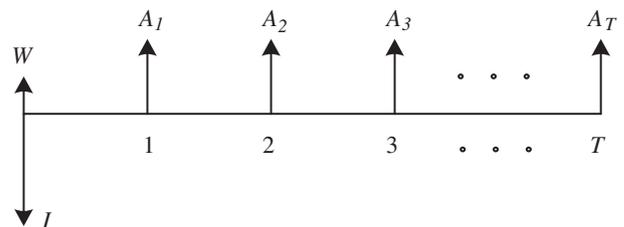


Fig. 2. Cash flow diagram of the DG investment project.

$$RoR_{DG,i} \geq RoR_D \quad (8.b)$$

In order to be sure that the constraints introduced in (4.b), (8.a), and (8.b) are all satisfied in optimization procedure, the *exterior penalty function method* is applied [44]. In this regard, we define

$$\begin{aligned} g_1 &= EENS - EENS_L \\ g_2 &= \sigma_{DG,i} - \sigma_L \\ g_3 &= RoR_D - RoR_{DG,i} \end{aligned} \quad (9)$$

Consequently the objective function changes as defined in (10).

$$\begin{aligned} f_1^{new} &= f_1 + \sum_{i=1}^3 pf_i \times G_i \\ f_2^{new} &= f_2 + \sum_{i=1}^3 pf_i \times G_i \\ f_3^{new} &= f_3 + \sum_{i=1}^3 pf_i \times G_i \\ f_4^{new} &= f_4 + \sum_{i=1}^3 pf_i \times G_i \end{aligned} \quad (10)$$

$$G_i = \max[0, g_i] \quad \forall i \in [1 - 3]$$

In which  $pf_i$  should be large enough so that all these constraints are satisfied.

## 2.2. Uncertainties modeling

Uncertainty is one of the imperative factors generally escalates the risk of any decision makings. Due to numerous uncertainties in power systems, such as in the system load growth, regulations, the decommissioning of generating units, technologies, and the market energy prices, the system analysis needs to be equipped with a well-organized method to efficiently get through these uncertainties. The load growth prediction and the electrical energy price variations in a competitive environment are accounted as the key factors allied with the discussed DG planning problem in power distribution systems. So, many ballpark methods have been applied to have them favorably handled in power system studies [43,45].

Monte Carlo simulation (MCS), analytical methods, and approximate methods can be accounted as the three main approaches of probabilistic modeling in power system studies. The MCS has been commonly brought into play to precisely model the uncertainties [45]. This method uses some deterministic routines to solve the problem in each step of simulation. The immense numbers of simulations are requisite to conquer the problem which deemed to be the main shortcoming of the MCS method. Analytical methods necessitate some mathematical assumptions in order to decipher the problem. In this category, some different methods such as the multi-linear model [45], the cumulant and Von Mises functions [46], and the Gram–Charlier expansion method [47] can be pointed out as well. Approximate methods employ the statistical properties of output random variables as the final result of a probabilistic analysis. Amongst is the First-Order Second-Moment Method (FSOMM) that has some disadvantages one of which is the requirement to evaluate the derivatives with respect to random variables [47,48]. The PEM has been recently grown up as an efficient method to handle the existent uncertainties [47,48]. The reason lies in its magnificent advantages as follows.

- The PEM uses deterministic routines in different stages of problem solving procedure similar to the MCS method, but with a much lower computational effort, in contrast.

- As the random input variables are approximated with their first three moments, the lack of data information is incapable of impeding the PEM effectiveness.

## 2.3. The PEM algorithm

The PEM was introduced by the Rosenblueth in 1975 for the moment's random quantity calculations and was later revisited in 1981 [47]. The PEM is applied to compute the moments of random variables  $Y$  which are a function of  $n$  random input variables  $X$ , i.e.,  $y = F(x_1, x_2, \dots, x_n)$ . These statistical moments are the mean (i.e., the first moment around the source) and the variance (i.e., the second moment with reference to the mean). In the DG placement problem, the input and output random variables can be defined as shown in (11) and (12), respectively.

$$X = [P_D, MP] \quad (11)$$

$$Y = f(X) = [RoR, EENS, CF_{DG,i}] \quad (12)$$

The statistical information of each input random variable is concentrated on  $K$  points, identified as concentrations, which are made available by the first three moments of input random variables. The number of concentrations identifies different variant of PEM which is called  $K$ -PEM [49–52]. In this paper, due to the required accuracy as well as the computational burden, 2-PEM is employed to model the addressed uncertainties in power distribution systems. Paying attention to these points and the relation function of the input and output variables, the statistical data concerning the output random variables can be obtained. The following steps deals with the algorithm of Probabilistic Optimal Power Flow (POPF) on the basis of 2-PEM [49].

- (1) A suitable Probability Distribution Function (PDF) is assigned to each probabilistic variable, including load and generation bids.
- (2)  $E(Y) = E(Y^2) = 0$ .
- (3) The essential parameters of the 2-PEM have to be determined through 13.a, 13.b, 13.c.

$$\zeta_{k,1} = +\sqrt{n} \quad (13.a)$$

$$\zeta_{k,2} = -\sqrt{n} \quad (13.b)$$

$$P_{k,1} = P_{k,2} = \frac{1}{2n} \quad (13.c)$$

where  $\zeta_{k,1}$ ,  $\zeta_{k,2}$ ,  $P_{k,1}$ , and  $P_{k,2}$  respectively denote the locations and probabilities of concentrations.

- (4)  $k$  is set to be one ( $k = 1$ ).
- (5) The concentrations ( $x_{k,1}$  and  $x_{k,2}$ ) have to be determined and the deterministic OPF has to be conducted employing the input vector  $X$ .

$$x_{k,1} = \mu_{X,k} + \zeta_{k,1} \cdot \sigma_{X,k} \quad (14.a)$$

$$x_{k,2} = \mu_{X,k} + \zeta_{k,2} \cdot \sigma_{X,k} \quad (14.b)$$

$$X = [\mu_{k,1}, \mu_{k,2}, \dots, x_{k,i}, \dots, \mu_{k,n}] \quad i = 1, 2 \quad (14.c)$$

- (6)  $E(Y)$  and  $E(Y^2)$  have to be then updated.

$$E(Y)^{(k+1)} \cong E(Y)^{(k)} + \sum_{i=1}^2 P_{k,i} \cdot h(X) \quad (15.a)$$

$$E(Y^2)^{(k+1)} \cong E(Y^2)^{(k)} + \sum_{i=1}^2 P_{k,i} \cdot h^2(X) \quad (15.b)$$

- (7)  $k = k + 1$  should be adopted and steps 5 and 6 have to be repeated until completed for all random input variables.  
 (8) Standard deviation and the expected value of  $Y$  need to be calculated through (16.a) and (16.b), respectively.

$$\mu_Y = E(Y) \quad (16.a)$$

$$\sigma_Y = \sqrt{E(Y^2) - \mu_Y^2} \quad (16.b)$$

### 3. Multi-objective optimization modeling

As have been discussed in the previous section, different key criteria should be considered in the attractiveness evaluation of the DG planning schemes. The conflictive behavior of these factors necessitates a well-organized approach to well deal with them all together. The following subsections aim to introduce a widely proven MO optimization method which can easily handle the noted problem.

#### 3.1. The NSGAI method

Multi-Objective (MO) optimization has been introduced as an advantageous tool to handle different contradictory, supportive or mathematically unrelated objective functions. There are two general approaches which are commonly utilized to deal with a MO optimization problem. The first one, i.e., the Priori methods, tries to solve the MO problem by combining and converting it into a single-objective optimization problem using any kinds of weighted sum techniques; the other one, the Posterior methods, organizes the MO optimization problem via non-dominancy concept [38,48]. Being rendered inoperative in the view of guaranteeing to find global Pareto optimal solutions and moreover, dependency to a prior knowledge about decision maker's preferences for combining objective functions, can be accounted as the main shortcomings of the priori methods [48]. In contrast, the posterior methods lead to Pareto optimal solutions employing non-dominancy concept which can give the decision makers a better estimate of MO optimization problem.

The optimization technique which is used to be frequent in solving MO optimization problems is referred to as non-dominated based Genetic Algorithm. Its applicability and robustness in MO optimization problems have been widely proved. However, it is criticized for its computational complexity, lack of elite population and diversity of final solutions. Moreover, the algorithm is much sensitive to the optimal parameter value, i.e., sharing parameter ( $\sigma_{Sharing}$ ). Hence, some errors are inevitable and therefore, this can lead to a miss of some optimal solutions [38]. As an improvement to bypass these deficiencies, a modified version, as a Fast Elitist Multi-Objective NSGAI, has been proposed in which there is no need to determine the sharing parameter, and this eventuates in a better sort and elitism to approach the final solution [38]. The step by step algorithm of NSGAI and its superiorities are explained in the following.

In NSGAI, the first population is initialized and sorted with respect to the objective function based on the non-dominancy concept in each front. Each Pareto front and its individuals are then assigned a rank due to the non-dominancy criterion, i.e., first front and its individuals which dominate on the others get rank 1; the second front which dominates on the others except the first front gets rank 2 and so on. The Pareto fronts members with the same non-dominancy rankings are then assigned a distance considering

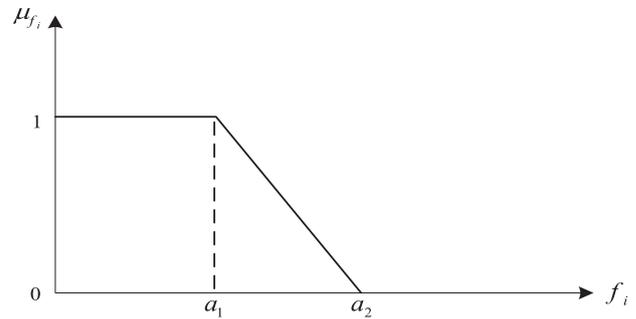


Fig. 3. The assigned fuzzy membership function to the objective functions.

Crowding Distance approach with regard to other members in the same Pareto front [38]. As the individual distances are determined, the parent populations are selected applying binary tournament algorithm. The binary tournament algorithm selects the parents based on the less non-dominancy rank value and more distance rank value. As a notification, the distance value is assigned to individuals in the same Pareto front. Consequently, the offspring populations are generated employing the traditional crossover and mutation operations. At the end, parents and offspring are composed to form a collection and next generation is selected from this collection. This process continues for next generations until the termination criterion (number of iterations) is satisfied.

For further information about the NSGAI algorithm and its procedure, one can refer to [38]. In this paper, NSGA II based on PEM method has been used in which selection and comparison are done using probabilistic criteria (i.e., assessment of expected value of fitness functions).

#### 3.2. Final decision making method

Since the NSGAI method favors the schemata of the Pareto optimum regions, and to accomplish the goal of their ranking, non-dominancy concept is applied over the whole population [38]. Upon finding the non-dominated set, a realistic and favorable solution needs to be found which can well compromise between different objectives [38,48]. In response, an efficient method should be employed to find the final optimal and practical solutions among those obtained solutions through the NSGAI. Many methods and solutions exist for the purpose. Amongst, the fuzzy approach is selected due to its simplicity and similarity to the human beings interpretation [38,48]. Fuzzy satisfying method is incorporated to handle the fuzzy sets, defined through membership functions and find the final solution. In fuzzy satisfying method, for each objective, a rigorous monotonically declining and continuous membership function is assigned. Regarding the minimum optimization problem, the preferred membership function is valued 1 at the minimum and 0 at the maximum of the objective function [38] (see Fig. 3).

The membership function value interprets the fact that to what extent a solution is approaching toward the objective function fulfillment. Then, it is time to solve the optimization problem, introduced in (17), and in accordance with the desired level previously defined by the decision maker, i.e.,  $\mu_{di}$ . Subsequently, the final optimal solution will be found [48].

$$\min_{X \in \text{Solutionset}} \sum_{i=1}^3 |\mu_{di} - \mu_{fi}(X)|^n \quad n \in [1, \infty) \quad (17)$$

As can be easily traced, membership function deviation of  $X$  would be minimized in comparison with the favorable level. The nuance in larger values of  $n$  would eventuate in a less sensitivity to the desirable level.

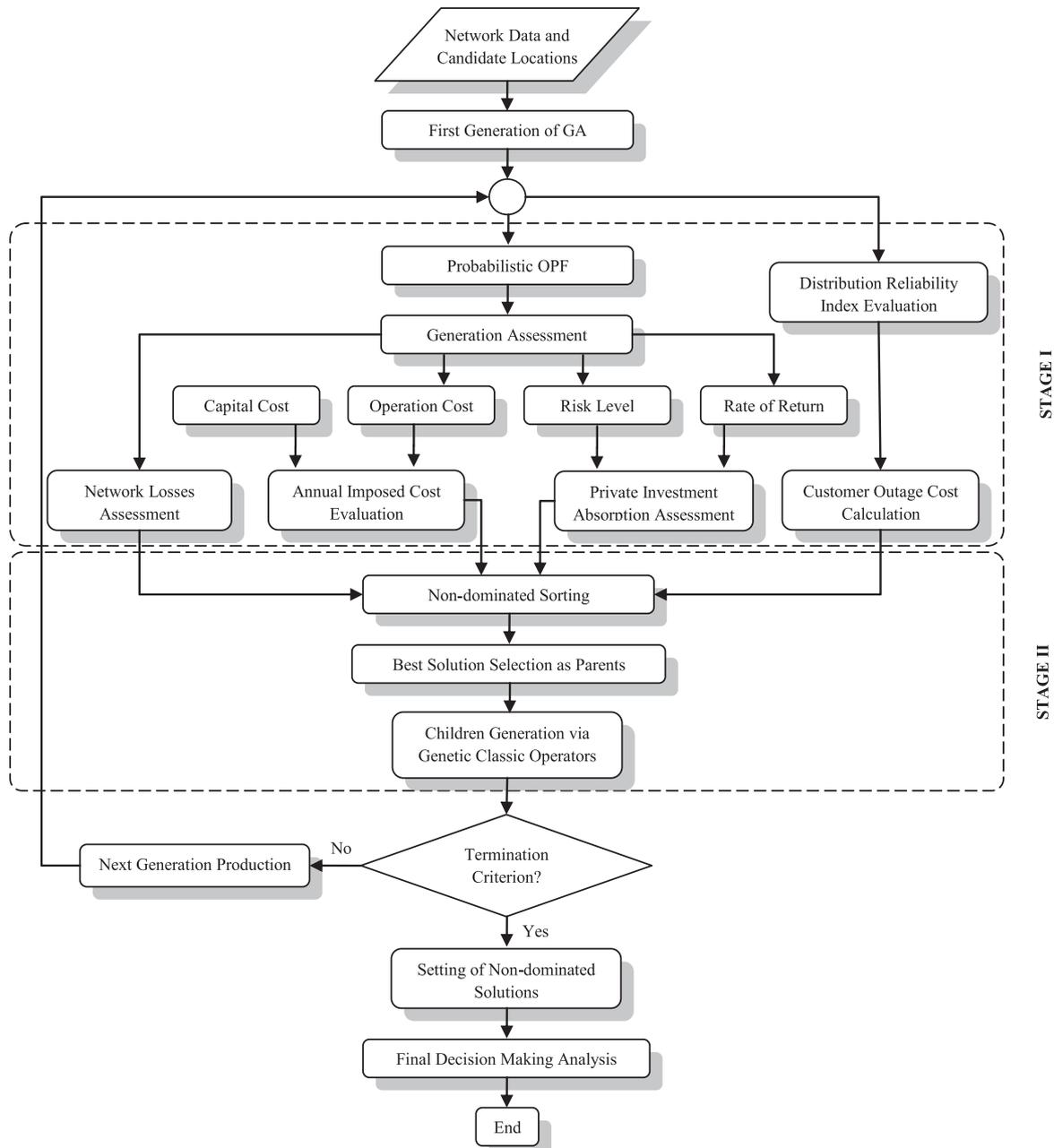


Fig. 4. The proposed scheme for DG placement.

#### 4. Proposed algorithm

The proposed algorithm of DG placement procedure in power distribution systems is presented in Fig. 4, in line with the previously-introduced concepts. As it has been delineated in this figure, the candidate buses to be considered for DG installation are initially determined through a sensitivity analysis. In so doing, it studies the impacts of DG penetration for each bus on the total loss and customer outage cost of the distribution network. Applying the GA, the first population is created which represents both the selected buses for DG installation and their capacities (candidate solutions).

The POPF is then performed to calculate the total loss and operation cost of the distribution network in presence of DG units. The annual cost including investment and operation costs

are then calculated through (1). Also, the rate of return and risk level of each candidate solution are the other factors of interest obtained by POPF. These factors are representatives of the project attractiveness in a private investor's point of view and are involved in the private investment absorption objective function through (8.a) and (8.b). The EENS index can be reached which accordingly eventuates in the customer outage cost. Having completed the first stage, the NSGAI evaluates the solutions taking the non-dominancy concept into consideration. If the termination criterion (iteration number of GA) is satisfied, the final decision making method (fuzzy satisfying method) is used to catch the final optimal solution out of the optimal Pareto solutions. Else, the GA classic operators have to be employed to generate the next generation. The second stage is completed by then.

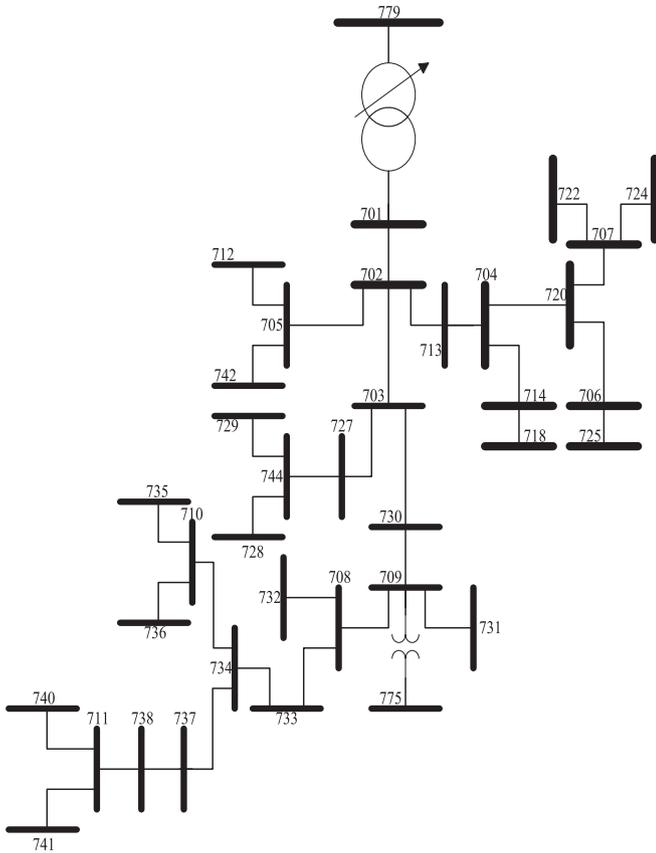


Fig. 5. The IEEE 37-bus test system under study.

## 5. Case study

### 5.1. Test system under study and the technical assumptions

The test network used for the verification of the proposed algorithm is the IEEE 37-bus test feeder, whose single line diagram is as depicted in Fig. 5. The distribution system includes 25 load points accompanied by a main substation of 2500 kV A. The substation is expected to connect the remainder of the system to the main grid [53]. The interested readers are referred to [53] for more information on this network. The maximum demand at the end of the planning horizon is forecasted to be 4500 kW with a forecasting error tantamount to 15%. It is assumed that the Disco has to set a 10-year plan horizon to meet the peak demand. Moreover, it is supposed that the Disco aims to provide at least 25% of the required electrical energy via DG technologies. The here-used technology is micro-turbine with the assumed characteristics presented in [14]. The operation and capital costs associated with this DG technology are assumed to be 6.09 cents/kWh and 750 \$/kW, respectively, and the plant factor is considered to be 55% [14]. The electricity market price is considered to be 0.06 \$/kWh with a 20% standard deviation. During this time period, the rate of inflation, discount, and load growth are respectively assumed to be 9%, 4%, and 6% [14]. The different types of customers considered here are presented in the Appendix. The outage costs associated with the customers in different load points are considered based on [54].

To decide where to have the DGs installed (candidate buses), a sensitivity analysis is performed. The sensitivity of the total network losses and the customer outage cost variations with respect to the net power injected into each bus, i.e.,  $\partial Loss/\partial P_{net,i}$  and  $\partial COC/\partial P_{G,i}$ , are investigated through (18) and (19). Final candidates would be then prioritized by (20).

$$LS_i = \frac{TL_{base} - TL_i}{TL_{base}} \quad (18)$$

$$COCS_i = \frac{COC_{base} - COC_i}{COC_{base}} \quad (19)$$

$$S_i = \alpha \cdot LS_i + \beta \cdot COCS_i \quad (20)$$

where  $\alpha$  and  $\beta$  are considered to be both equal to 0.5. It is noteworthy that these two parameters can be determined in accordance with the Disco's long term policies and also can reflect the technical aspects of the understudied feeder. The sensitivity factors for different buses which are shown in Fig. 6 can effectively help the decision maker to select the appropriate candidates for DG installation. As can be seen in this figure, load points 724, 725, 740, and 741 are the most critical buses and thus can be the apt locations to be selected for DG placement. Among all the network buses, 14 critical ones are selected as the proper candidates.

### 5.2. Results and discussions

The proposed algorithm was implemented in MATLAB environment and found 48 optimal solutions with the population size of 120 and passing 100 iterations. It took each iteration on average 81 s to run. The noteworthy is that the proposed approach in this paper is considered as a planning-based, but not an operational-base decision making. As a result, regarding the placement procedure as a decision making in the planning horizon, timing issues and the execution times of the presented scheme are not the concern. Moreover, referring the relevant literature on the optimization algorithms, it can be concluded that the NSGAI method is among the fastest ones which also ensures the global optima of the optimization problems [38].

This paper has employed the conventional OPF solvers for this study and uses the MATPOWER function in MATLAB environment. This function has been modified to be applied for distribution studies. This package uses a proven and robust optimization method to find the optimal result of the OPF problem, i.e., interior point method. As a notification, the authors have run these studies with the OPENDSS package which is designed for distribution studies and the same results have been obtained too.

The non-dominated solutions can be found in Figs. 7a–7d. These figures well present the trade-off regions for different objectives and provide the planner with some valuable information on how to decide about the optimal distribution system reinforcement policies.

As shown in Figs. 7a–7c, the annual imposed costs of optimal solutions varies between 422 k\$ and 856 k\$ as the customer outage cost changes in the range of 8.04–13.722 k\$/year, the absorbed private investment varies between 25.643 k\$ and 1072.589 k\$, and the distribution feeder total loss changes between the 196.49 kW and 301.79 kW.

More comprehensively speaking, if the annual investment of an assumed Disco is limited to, for example, 500 k\$, the changes in customer outage cost of the optimal solution is about 2.150 k\$/year. Compare this variations with those of annual imposed costs in the range of 700–800 k\$ in which the customer outage cost changes is about 0.900 k\$/year. This comparison shows that the investment cost more than a threshold value would be unable to treat as a remedy to the imposed customer outage cost. Such analysis gives a better insight to a Disco manager in dealing with the investment sensitivity to the system reliability.

As depicted in Fig. 7b, the absorbed private investment grows as the annual imposed costs increase (more new DG units are employed). However, this trend saturates as the annual imposed costs meet the knee point of 800 k\$. It is just due to the decrease in the

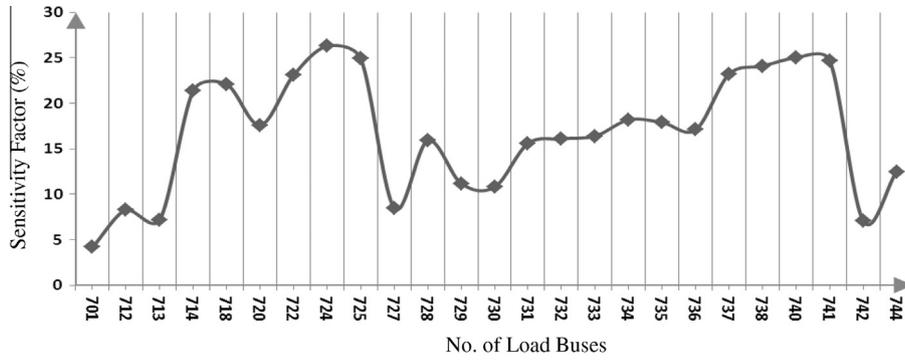


Fig. 6. Sensitivity factor of the network buses.

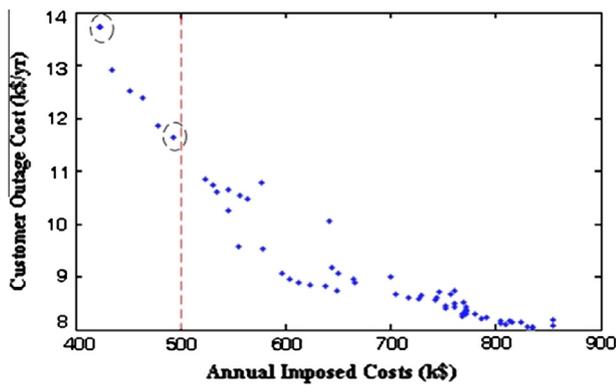


Fig. 7a. The trade-off between the customer outage cost and annual imposed cost.

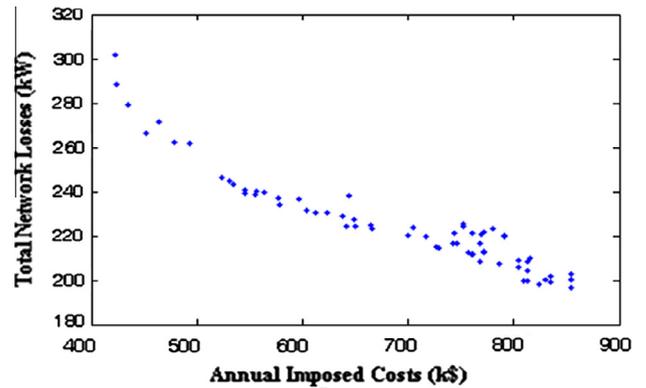


Fig. 7c. The trade-off between the total network losses and annual imposed cost.

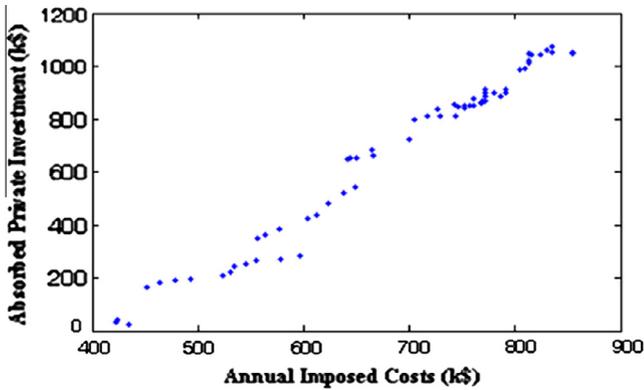


Fig. 7b. The trade-off between the absorbed private investment and annual imposed cost.

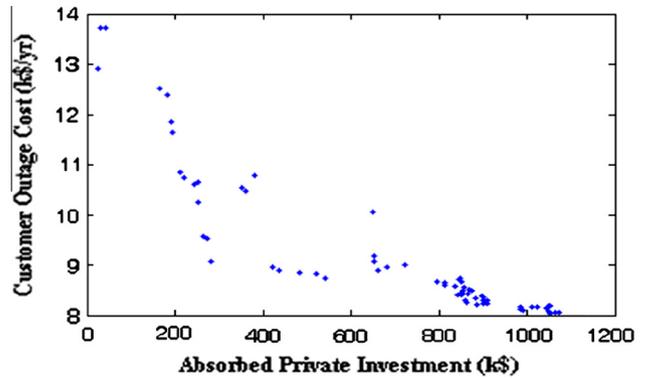


Fig. 7d. The trade-off between the customer outage cost and absorbed private investment cost.

rate of return associated with more penetration rate of DG units. Even though saturation occurred in the trend of absorbed private investment, it is of great importance to consider that more investment leads to harvesting more benefits in total loss reduction or customer outage cost improvements.

Fig. 7c delineates the trade-off between the annual imposed cost and total network losses. Similar to the recent discussions, total network losses follow a descending trend in terms of annual imposed costs of DG units. This result can be well justified so that the sensitivity of the total network losses to the more penetration of DG units decrease as the annual imposed costs increase. This fact is shown in Fig. 8. As inferred, some fluctuations can be traced in some DG penetration levels (some optimal solutions in the first

Pareto front). To put a figure on this feature, conduct an attention on the 5.4%, 5.8%, and 5.9% penetration levels of DG units. The descending trend of total network losses has seen an unpredicted increase at the penetration level of 5.9%. This unforeseeable happening is justified regarding the contradictory characteristics of the other objectives which create the plans virtually seem unreasonable if taking only total network losses objective into consideration. However, the fact is hidden through the former explanations.

This DG penetration increment (5.9% vs. 5.8%) maybe the result of customer outage cost improvement. Put in mind the fact that facing with MO optimization problems calls for a more thorough attention to the objective functions all together in the analysis of

the optimal plans. The trade-off region shown in Fig. 7d is associated with the absorbed private investment vs. customer outage cost which follows the same attributes as those in Fig. 7a.

### 5.3. Final plan selection

Up to now, the trade-off regions between different objectives in the first Pareto front have been reported and their various impressive attributes have been explored. But, it should be considered that in the viewpoint of a Disco manager, there is no difference between all the optimal plans. This necessitates applying an efficient decision making method to meet the Disco's requirements oriented from the company's long term polices.

As introduced in Section 3, the fuzzy satisfying method has been found to be a well-organized approach to model the human needs into a mathematical manner. In this respect, applying the fuzzy satisfying method leads to the final optimal plan being reached as demonstrated in Table 1, where in three final optimal plans with different satisfactory levels are offered. The  $\mu_{d1}$  to  $\mu_{d4}$  respectively denote the satisfactory levels for the annual imposed cost, total network losses, absorbed private investment, and customer outage cost. As an assumption, a Disco may seek a plan which significantly takes the amount of imposed cost into consideration (a cost-based perspective). So, this objective should play a vital role in the final scheme obtained by the decision making method. In this context, this calls for a more considerate selection of the satisfactory levels ( $\mu_{d1}$  to  $\mu_{d4}$ ). As shown in the first plan in Table 1, the obtained results may not satisfy the Disco's technical goals, i.e., total network losses, and customer outage cost. To investigate the sensitivity of the final optimal plan to the amount of the annual imposed cost, the planner selects the second proposal for the objective functions, as shown in detail in Table 1. In these new conditions, the annual imposed cost has raised about 32 k\$ which outcomes a decrease of respectively 285.04 kW and 710 \$/year in the network losses and customer outage cost. The third case in Table 1 reports a situation where the planner decides to scrutinize whether it is reasonable to

rely on the private investment in the DG planning decision of Disco or not. As it is implied from the results in this case, not only the annual imposed costs have experienced a growth equivalent to 19 k\$, but also the absorbed private investment has increased by 11 k\$. In comparison with the second case, the percentage of the private sector contribution in the required investment of DG penetration has been much increased (as it is anticipated due to  $\mu_{d3}$  increment from 0.8 to 0.9).

### 5.4. Further explorations

The authors believe that the proposed approach, as a DG placement algorithm, is comprehensive and also generic enough to be applied in real and practical applications as well as in the cases where there are different types of DG technologies involved. The reason lies in the fact that by considering the effective objectives and probabilistic handling of the problem, the optimization basis can be found as practical as possible. Moreover, the NSGAI optimization technique has been proved to be capable of handling the large-scale optimization problems [38,48,55]. As a result, any concern on the scalability of the proposed approach would be obviated.

The proposed method in this paper is general enough to be applied for various technologies of DGs (renewable or the other ones) with a little modifications. For example, a wind turbine can be accepted as the DG technology. So, the only part that should be modified in the paper is modeling its output and considering its little operation cost in comparison with the micro-gas turbines. So, the stochastic model of wind turbine output can be fed as some input random variables into the PEM procedure. It would be consequently possible to run the proposed method with this modification and obtain the Pareto results of the DG placement problem again. The proposed method can also open a window for comparing various DG technologies in the characteristics of final optimal results and can give the decision maker a more practical viewpoint

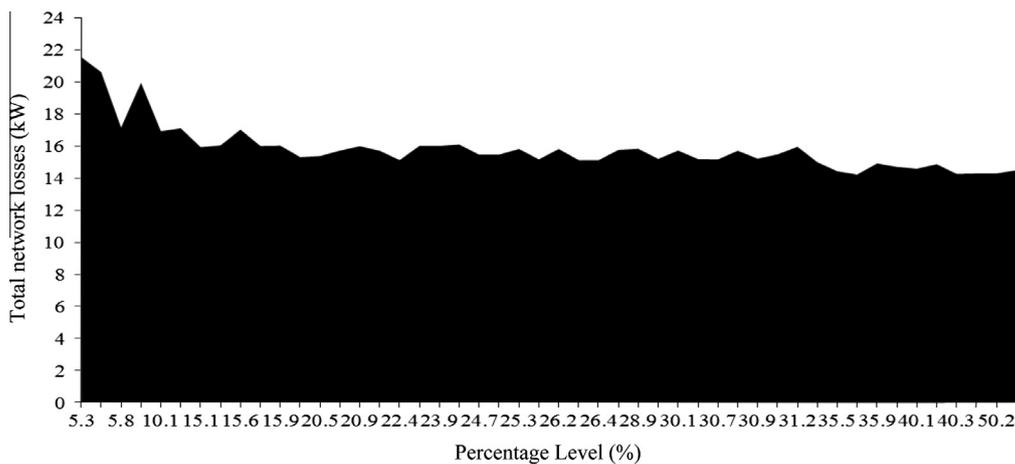


Fig. 8. Total network losses variation by different penetration levels of distributed generation.

Table 1  
Different satisfactory levels and final candidate buses.

Satisfactory level				Objective function value				Candidate buses													
$\mu_{d1}$	$\mu_{d2}$	$\mu_{d3}$	$\mu_{d4}$	Obj. 1 (k\$)	Obj. 2 (kW)	Obj. 3 (k\$/year)	Obj. 4 (k\$)	714	718	720	722	724	725	728	733	734	735	737	738	740	741
0.8	0.6	0.6	0.8	545	239.26	10.245	254	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0.6	0.8	0.8	0.8	577	214.22	9.535	272	0	1	0	1	1	1	0	1	0	0	0	1	1	0
0.5	0.8	0.8	0.9	596	216.74	9.056	283	1	0	1	1	1	1	1	0	1	0	0	0	0	1

**Table 2**

Test network buses and the assigned load types.

Load type	Load points
Ind. <sup>a</sup>	701, 720, 728, 732, 734, 737, 742
Res. <sup>b</sup>	712, 714, 722, 727, 730, 733, 740, 744
Com. <sup>c</sup>	713, 718, 724, 725, 729, 731, 735, 736, 738, 741

<sup>a</sup> Industrial load.<sup>b</sup> Residential load.<sup>c</sup> Commercial load.

to understand the differences between these technologies and their effects on the technical and financial concerns.

The other point to be emphasized is that the uncertainties discussed in the proposed probabilistic method in this paper are the amount of predicted loads at various buses and electricity price as well. It should be mentioned that the number of these uncertainties do not have anything to do with the time duration of studies and also with the number of buses. So, one can conclude that the concentration points of the random variables will not have such a high dimension that hurts its validity. As a complementary to this discussion, this attribute of the proposed scheme can be claimed as one of its superiorities up on the other methods.

## 6. Conclusion

In this paper, something worthwhile was accomplished, i.e., the presentation of a new MO approach to decide about the optimal locations of DGs in power distribution systems. The offered method would be able to well meet the power system requirements of nowadays deregulated environment and is appreciated in handling power distribution system technical and economic challenges. Of the main pivotal features of this paper is contributing the private investment requirements in the DG placement plans. This feature, as well as the network losses reduction, system reliability improvement, and total imposed cost consideration could provide the Discos' managers with some valuable information associated with their planning policies. Moreover, a robust probabilistic approach, i.e., PEM, was employed to accurately and less-computationally model the unavoidable uncertainties in power systems. The offered MO problem was managed via the NSGAI approach whose applicability in the cases of MO problems is widely proved. In last, fuzzy satisfying method gave the algorithm a hand to find the final optimal solutions. The proposed method was applied on the IEEE 37-bus test feeder and its effectiveness was expectedly approved. It was concluded that the optimal DG locations could be obtained through the trade-off between the addressed objective functions, and would be the most reliable solutions.

## Appendix A. Appendix

This section is to introduce the customer types associated with each load point. Hence, the load categories together with their assigned buses are demonstrated in Table 2.

## References

- Jenkins N, Allan R, Crossley P, Kirschen D, Strbac G. Embedded generation. 1st ed. UK: Cambridge University; 2008.
- Borbely A, Kreider JF. Distributed generation, the power paradigm for the new millennium. New York: CRC Press; 2001.
- Wood J. Local energy: distributed generation of heat and power. London: IET; 2008.
- Zerriffi H. Rural electrification strategies for distributed generation. New York: Springer; 2010.
- Moreno-Munoz A et al. Improvement of power quality using distributed generation. *Int J Electr Power Energy Syst* 2010;32:1076–609.
- Chowdhury SP, Chowdhury S, Crossley PA. Islanding protection of active distribution networks with renewable distributed generators: a comprehensive survey. *Electr Power Syst Res* 2009;79:984–92.
- Xi C, Wenzhong G. Effects of distributed generation on power loss, loadability and stability. *Southeastcon* 2008:468–73.
- Gautam D, Mithulananthan N. Optimal DG placement in deregulated electricity market. *Electr Power Syst Res* 2007;77:1627–36.
- Hung D, Mithulananthan N, Bansal R. Multiple distributed generators placement in primary distribution networks for loss reduction. *IEEE Trans Ind Electr* 2011.
- Viral R, Khatod DK. Optimal planning of distributed generation systems in distribution system: a review. *Renew Sustain Energy Rev* 2012;16:5146–65.
- Jen-Hao T, Tain-Syh L, Yi-Hwa L. Strategic distributed generator placements for service reliability improvements. In: *Proceedings of the IEEE power engineering society summer meeting*; 2002. p. 719–24.
- Hemdan NGA, Kurrat M. Efficient integration of distributed generation for meeting the increased load demand. *Int J Electr Power Energy Syst* 2011;33:1572–83.
- Khalesi N, Rezaei N, Haghifam MR. DG allocation with application of dynamic programming for loss reduction and reliability improvement. *Int J Electr Power Energy Syst* 2011;33:288–95.
- Zangeneh A, Jadid S. Fuzzy multiobjective model for distributed generation expansion planning in uncertain environment. *Euro Trans Electr Power* 2009;21:129–41.
- In-Su B, Jin-O K, Jae-Chul K, Singh C. Optimal operating strategy for distributed generation considering hourly reliability worth. *IEEE Trans Power Syst* 2004;19:287–92.
- Borges CLT, Falcao DM. Optimal distributed generation allocation for reliability, losses and voltage improvement. *Int J Electr Power Energy Syst* 2006;28:413–20.
- Karaki SH, Kayssi AI, Karaka HS. Capacitor placement for switching noise reduction using genetic algorithm and distributed computing. *Electr Eng* 2005;87:11–88.
- Kim KH, Lee YJ, Rhee SB, Lee SK, You SK. Dispersed generator placement using fuzzy-GA in distribution systems. *IEEE Power Eng Soc Summer Meet* 2002;3:1148–53.
- Lakshmi Devi A, Subramanyan B. Sizing of DG unit using genetic algorithms to improve the performance of radial distribution systems. *Int J Electron Electr Eng* 2010;13:48–55.
- Moradi MH, Abedini M. A combination of genetic algorithm and particle swarm optimization for optimal DG location and sizing in distribution systems. *Int J Electr Power Energy Syst* 2012;34:66–74.
- Saedighizadeh M, Rezaadaeh A. Using genetic algorithm for distributed generation allocation to reduce losses and improve voltage profile. *World Acad Sci Eng Technol* 2008;3(7):251–6.
- Kuri B, Redfem MA, Li F. Optimization of rating and positioning of dispersed generation with minimum network disruption. *IEEE Power Eng Soc General Meet* 2004;2:2074–8.
- Celli G, Ghiani E, Mocci S, Pilo F. A multi-objective evolutionary algorithm for the sizing and siting of distributed generation. *IEEE Trans Power Syst* 2005;20:750–7.
- Phonrattanasak P. Optimal placement of DG using multi objective particle swarm optimization. In: *Proceedings of the 2nd international conference on mechanical and electrical technology, ICMET*; September 2010. p. 342–6.
- Lingfeng W, Singh C. Reliability-constrained optimum placement of reclosers and distributed generators in distribution networks using an ant colony system algorithm. *IEEE Trans Syst Man Cybern C: Appl Rev* 2008;38(November 6):757–64.
- Dasan SGB, Ramalakshmi SS, Devi RPK. Optimal siting and sizing of hybrid distributed generation using EP. In: *Proceedings of the international conference on power systems, ICPS*; December 2009. p. 1–6.
- Rotaru F, Chicco G, Grigoras G, Cartina G. Two-stage distributed generation optimal sizing with clustering-based node selection. *Int J Electr Power Energy Syst* 2012;40:120–9.
- Nara K, Ikeda K, Hayashi Y, Ashizawa T. Allocation of Tabu search to optimal placement of distributed generators. *IEEE Power Eng Soc Winter Meet* 2001;2:918–23.
- Falaghi H, Haghifam MR. ACO based algorithm for distributed generations sources allocation and sizing in distribution systems. In: *Proceedings of IEEE Lausanne power technology. University of Tehran*; 2007. p. 555–60.
- El-Zonkoly AM. Optimal placement of multi-distributed generation units including different load models using particle swarm optimization. *IET Gener Transm Distrib* 2011;5(7):760–71.
- Haghifam MR, Falaghi H, Malik OP. Risk based distributed generation placement. *IET Gener Transm Distrib* 2008;2:252–60.
- Buayai K. Optimal multi-type DGs placement in primary distribution system by NSGA-II. *Res J Appl Sci Eng Technol* 2012;4(19):3610–7.
- Deb K. Multi-objective optimization using evolutionary algorithms. New York: Wiley; 2003.
- Mendoza F, Bernal-Agustin JL, Dominguez-Navarro JA. NSGA and SPEA applied to multi-objective design of power distribution systems. *IEEE Trans Power Syst* 2006;21(4):1938–45.
- Shen F, Yang X. Optimization algorithm of urban road traffic signal plan based on NSGAI. In: *Proceedings of international conference on intelligent computation technology and automation*; 2008. p. 398–401.

- [36] Golea NE-H, Melkemi KE, Melkemi M. A novel multi-objective genetic algorithm optimization for blind RGB color image watermarking. In: Proceedings of 7th international conference on signal-image technology and internet-based systems (SITIS); 2011. p. 306–13.
- [37] Sindhya K, Miettinen K, Deb K, A hybrid framework for evolutionary multi-objective optimization, *IEEE Trans Evolut Comput*, in press. <http://dx.doi.org/10.1109/TEVC.2012.2204403>.
- [38] Deb K, Pratap A, Agarwal S, Meyarivan T. A fast elitist multi-objective genetic algorithm: NSGA-II. *IEEE Trans Evolut Comput* 2002;6(April 2):182–97.
- [39] Singh AK, Parida SK. Selection of load buses for DG placement based on loss reduction and voltage improvement sensitivity. In: Proceedings of international conference on power engineering, energy and electrical drives; 2011.
- [40] Billinton R, Allan RN. Reliability evaluation of power systems. 2nd ed. New York: Plenum Press; 1992.
- [41] Markowitz H. Portfolio selection. *Journal of Finance* 1952;77–91.
- [42] Lee CW, Ng SKK, Zhong J. Portfolio optimization in transmission investment in deregulated market. In: Proceedings of the IEEE power engineering society general meeting; June 2007. p. 1–8.
- [43] Allan RN, Leite da Silva AM. Probabilistic load flow using multi linearizations. *Proc Inst Electr Eng C: Gener Transm Distrib* 1981;128(5):280–7.
- [44] Yeniay O. Penalty function methods for constrained optimization with genetic algorithms. *Math Comput Appl* 2005;10:45–56.
- [45] Allan RN, Al-Shakarchi MRG. Probabilistic techniques in ac load-flow analysis. *Proc Inst Electr Eng*. 1977;124(February 2):154–60.
- [46] Sanabria LA, Dillon TS. Stochastic power flow using cumulants and von Misses functions. *Int J Electr Power Energy Syst* 1986;8:47–60.
- [47] Erbic G, Canizares CA. Probabilistic optimal power flow in electricity markets based on a two-point estimate method. *IEEE Trans Power Syst* 2006;21:1883–93.
- [48] Maghouli P, Hosseini SH, Oloomi M, Shahidehpour M. A multi-objective framework for transmission expansion planning in deregulated environments. *IEEE Trans Power Syst* 2009;24(May 2):1051–61.
- [49] Chun-Lien Su. Probabilistic load-flow computation using point estimate method. *IEEE Trans Power Syst* 2005;21(4):1843–51.
- [50] Xiaomeng A, Jinyu W, Luo Weihua W. A discrete point estimate method for probabilistic load flow based on the measured data of wind power. *IEEE Innov Smart Grid Technol – Asia (ISGT Asia) 2012(21–24 May):1–6*.
- [51] Xialing X, Tao L, Xiaoming Z. Probabilistic analysis of small signal stability of microgrid using point estimate method. In: International conference on sustainable power generation and supply; 6–7 April 2009. p. 1–6.
- [52] Liu X, Zhong J. Point estimate method for probabilistic optimal power flow with wind generation. In: Int conf on electrical engineering (ICEE), Algeria; 2009.
- [53] Kersting WH. Radial distribution tests feeders. *IEEE Trans Power Syst* 1991;6(August 3):975–85.
- [54] Jakobsson M, Engblom O, Alvehag K. Representative test systems for Swedish distribution networks. In: Proceedings of the 20th international conference and exhibition on electricity distribution – Part 2, CIRED; June 2009. p. 8–11.
- [55] Maghouli P, Hosseini SH, Buygi MO, Shahidehpour M. A scenario-based multi-objective model for multi-stage transmission expansion planning. *IEEE Trans Power Syst* 2011;26(1):470–8.